A Generalized Approach to Wide-Band S-Parameter Extraction from FD-TD Simulations Applicable to Evanescent Modes in Inhomogeneous Guides

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***ABSTRACT* — In this paper we introduce a new method of S-parameter extraction from time domain electromagnetic simulations. The method can be effectively applied to wide-band analyses in a multi-mode environment including evanescent modes. We show that the new method must be based on a proper choice of the definition of the S-matrix, and we discuss somewhat unusual properties of such matrices for evanescent modes.**

I. INTRODUCTION

S-parameter extraction from FD-TD simulations of microwave circuits has been considered by many authors. Initially the interest concentrated on propagating modes with the assumption of perfect matching of the ports. Due to problems with wide-band simulation of well matched ports, more general methods assuming a controlled mismatch at the ports were further introduced [4]..[7]. Unfortunately, most of these methods still concentrate on propagating modes, while characterisation of multiport devices with evanescent modes at ports is also of great practical interest. It is well known that the most effective way of the analysis can often be obtained by circuit segmentation [2] and by calculating different segments separately applying the same method or even applying different simulation methods to different segments. As an example, FD-TD mode expanded results [3][4][7] can be used for further matrix operations or in conjunction with the mode matching method. However, for such applications we need a method of wide-band S-parameter extraction, applicable to any practical case including extraction of evanescent modes in a multi-mode environment at ports defined at inhomogeneously filled waveguides. To our knowledge, such a method has not been reported yet and we will try to fill this gap here.

It has to be noted that the earlier reported methods addressing evanescent modes assume that the field distribution across the port does not change with frequency. Therefore they are applicable only to ports defined at homogeneous waveguides [4]..[6], and cannot be directly extended to inhomogeneous waveguides. Admittedly, the method of [7] could in principle be extended to evanescent modes at inhomogeneous ports, but such a case has not been discussed in [7] and, moreover, would entail extensive numerical operations in the extraction of individual templates for each considered frequency.

In this paper we introduce a new method which is free of the above restrictions and inconveniences. We show that the new method must be based on a proper choice of the definition of the S-matrix, and we discuss somewhat unusual properties of such matrices for evanescent modes.

II. ON THE PROPER CHOICE OF S-MATRIX DEFINITION AND MODE EXTRACTION

The definition of S-matrix does not bring any uncertainty as far as the incident and reflected waves at ports can be defined with respect to a real reference impedance.

A necessity to extend the S-matrix definition to non-real reference was noticed several decades ago, and a solution best known and used up to now was proposed by Kurokawa [1]. Following his definition, the incident and reflected power waves are described as follows:

$$a_i = \frac{U_i + Z_{ref} I_i}{2\sqrt{\left|\mathrm{Re}(Z_{ref})\right|}} \qquad (1)$$

$$b_i = \frac{U_i - Z_{ref}^{*} I_i}{2\sqrt{\left|\mathrm{Re}(Z_{ref})\right|}} \qquad (2)$$

where U_i and I_i are the mode voltages and currents at the i-th port, Z_{ref} is the reference impedance and asterisk denotes the complex conjugate.

A basic problem with physical waves characterized by non-imaginary propagating constant (and by non-real characteristic impedance) is that the incident and reflected waves are not orthogonal. This causes that the power transmitted to a port is not equal to the difference between the power flows calculated separately for the incident and reflected waves. Kurokawa's power waves defined with respect to non-real Z_{ref} are artificially orthogonalized. As a result, they can form a basis for a consistent circuit theory but they do not describe correctly the general physical reality, in particular:

- incident and reflected power waves after (1) do not obey Maxwell equations
- reflection coefficient calculated with (1) is also different from the physical one
- moreover, the definitions of (1) are impractical for evanescent modes because $\mathrm{Re}(Z_i) \rightarrow 0$

Kurokawa's definition is not controversial when we assume a real reference impedance. His approach can even be applied to evanescent waveguide modes [5] provided that the waveguide port can be terminated by real impedance, which does not generate other modes. Thus, it is applicable only to homogeneous waveguides.

Another important problem is the choice of the definition of voltage and current associated with a particular waveguide mode, and their extraction in a multi-mode environment. A typical approach consists in calculating a scalar product (across the port) of the field $\boldsymbol{E}(x,y,t)$ obtained from the 3-D FD-TD simulation and the field $\boldsymbol{e}_{Ti}(x,y)$ of a pure i-th mode (mode template) obtained analytically or from 2-D FD-TD eigenfunction analysis [6]

$$U_i = \iint_s \mathbf{E}(x,y,t)\, \mathbf{e}_{Ti}(x,y)\; ds \qquad (3)$$

The approach described by (3) is adequate for homogeneous waveguides. In the case of inhomogeneous waveguides it can be proven ([13] Ch.10) that the modes are not orthogonal with respect to scalar products like (3), and thus this approach does not separate the modes correctly. In the quoted references only [7] uses vector products applicable to inhomogeneous lines.

III. The Proposed Method

1. We propose that the mode voltages and currents be defined in the form of integrals across the port reference plane z_p expressed as follows

$$U_i(t) = \iint_s \boldsymbol{E}(x,y,z_p,t) \times \boldsymbol{h}_{Ti}(x,y,w_T)\; d\boldsymbol{s} \qquad (4)$$

$$I_i(t) = \iint_s \boldsymbol{e}_{Ti}(x,y,w_T) \times \boldsymbol{H}(x,y,z_p,t)\; d\boldsymbol{s} \qquad (5)$$

where $\boldsymbol{e}_{Ti}(x,y,w_T)$ and $\boldsymbol{h}_{Ti}(x,y,w_T)$ are *E*- and *H*- field amplitudes of the i-th mode template at the frequency w_T while:

$$\iint_s \boldsymbol{e}_{Ti}(x,y,w_T) \times \boldsymbol{h}_{Ti}(x,y,w_T)\, d\boldsymbol{s} = 1 \qquad (6)$$

3. The S-matrix extraction is performed by an extension of the differential method originally reported in [9]. In the applied method we extract (from the port reference plane fields) the values of :

$$U_i(w) = F\{U_i(t)\}; \quad I_i(w) = F\{I_i(t)\}; \qquad (7)$$

$$U_i'(w) = F\left\{\frac{\partial U_i(z,t)}{\partial z}\right\}; \quad I_i'(w) = F\left\{\frac{\partial I_i(z,t)}{\partial z}\right\} \qquad (8)$$

for each i-th mode, where *F* denotes the Fourier transform.

2. We propose the following definition of the incident and reflected waves for S-matrix extraction:

$$a_i(w) = \frac{U_i(w) + Z_i(w) I_i(w)}{2\sqrt{|Z_i(w)|}} \qquad (9)$$

$$b_i(w) = \frac{U_i(w) - Z_i(w) I_i(w)}{2\sqrt{|Z_i(w)|}} \qquad (10)$$

with $Z_i(w)$ being the mode impedance equal to the ratio of the mode voltage and current in the reflection-less case. Let us note that following (4)(5), at the frequency w_T the mode impedance of the i-th mode is:

$$Z_i(w_T) = 1 \qquad (11)$$

Unlike Kurokawa's definitions, those presented above are consistent with the physical reality and in particular with Maxwell's equations. For example, reflection co-efficient in an infinitely long waveguide is always equal to zero, also below the cut-off frequency of the considered mode (which is not the case in Kurokawa's theory).

Using the quantities specified in (7) and (8), and following the rules of the differential method [9], we can extract:

$$Z_i(w) = \sqrt{\frac{U_i(w) U_i'(w)}{I_i(w) I_i'(w)}} \qquad (12)$$

and then the values of $a_i(w)$ and $b_i(w)$ using the relations (7)..(10). The quantities of (7) and (8) also allow to extract the propagation constant at the i-th port:

$$g_i(w) = a_i(w) + jb_i(w) = \sqrt{\frac{I_i'(w) U_i'(w)}{I_i(w) U_i(w)}} \qquad (13)$$

The knowledge of the propagation constant provides additional information for more accurate extraction of numerically calculated derivatives of (8).

Segments of a microwave circuit described by S-matrices defined in points 1..3 above can be subject to cascading

/de-cascading (or embedding/de-embedding) operations basically in the same way, regardless of the reference impedance being real, imaginary or complex. However, it should be noted that with a non-real mode impedance (unlike the real mode impedance case):

a) the power transmitted into the *i*-th port

$$P_i \neq a_i^2 - b_i^2. \quad (14)$$

b) in a lossless multi-port the summation of squares of S-matrix elements over one column or one row does not give unity

c) in a lossless reciprocal two-port $|S_{11}| \neq |S_{22}|$ (15)

The above approach has been developed by the authors and applied in FD-TD practice. Afterwards they have found that points 1 and 2 are consistent with the "true S-matrix" approach developed by Marks and Williams and described in their very interesting theoretical paper [8].

IV. Software Implementation and Example of Analysis

The method described in this paper has been implemented by the authors in the FD-TD solver [14]. The operations on S-matrices describing circuit segments have been performed with the S-matrix Converting Module prepared under the authors' supervision by T.Ciamulski [12].

As an example of the analysis, we consider a two-resonator H-plane filter in rectangular waveguide technology. The structure is air-filled except for a teflon bar in the coupling area between the resonators. Moving the bar closer to the waveguide center increases the coupling between the resonators and thus deepens a ripple in the center of the filter chracteristics of Fig.3. The side position of the bar produces a maximally flat filter characteristic. The shape of the filter is presented in Fig.1.

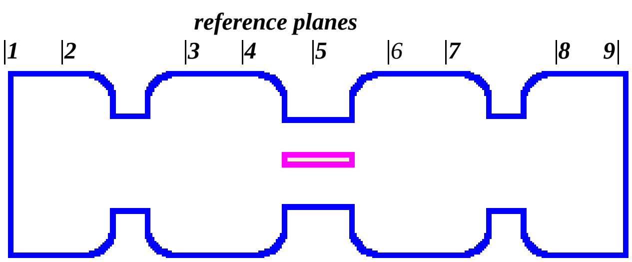

Fig.1. The shape of the considered 2-resonator waveguide H-plane filter with a teflon bar in the center.

In the first numerical experiment we compare the results of the FD-TD analysis performed on the entire structure (between reference planes 1 and 9), and on the two halves of the structure (between the planes 1-5 and 5-9) with subsequent S-matrix cascading. The results are presented in Fig.3. The differences between the two approaches are displayed on the separate amplified scale because they are extremely small (order of 0.002). It is also very interesting to look at the S-parameters of one half of the circuit (planes 1-5) presented in Fig.2. We can see that $|S_{11}|$=1 which was expected since in the reference plane 5 we have an evanescent mode transmitting no real power to the matched load. Other elements (S_{12}= S_{21} and S_{22}) are more difficult for intuitive interpretations because they adopt values well above unity in a frequency band close to the cavity resonance. However, in view of (14) and (15), such a behaviour is not contradictory to the theory and the cascading of partial results of Fig.2 produces correct filter characteristics as presented in Fig.3.

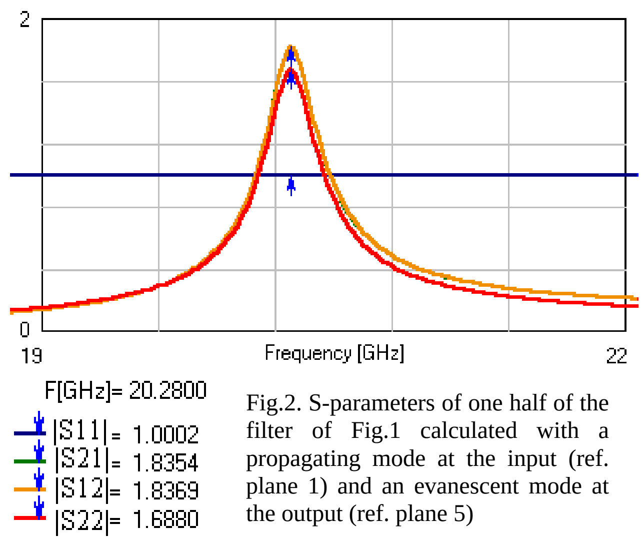

Fig.2. S-parameters of one half of the filter of Fig.1 calculated with a propagating mode at the input (ref. plane 1) and an evanescent mode at the output (ref. plane 5)

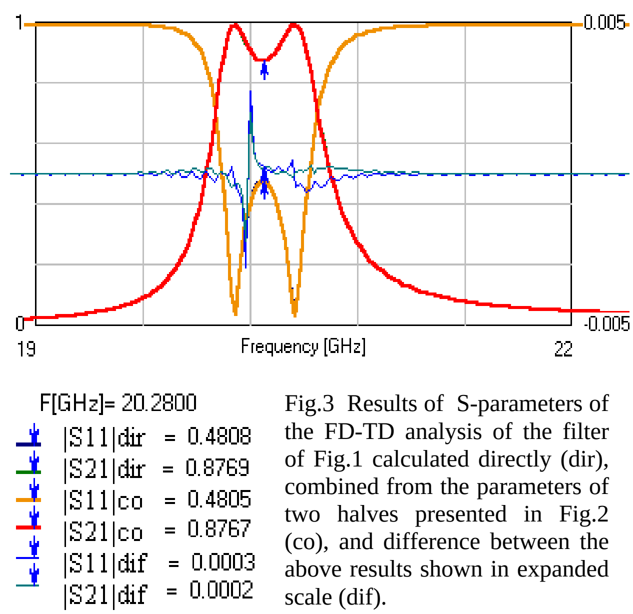

Fig.3 Results of S-parameters of the FD-TD analysis of the filter of Fig.1 calculated directly (dir), combined from the parameters of two halves presented in Fig.2 (co), and difference between the above results shown in expanded scale (dif).

In a further experiment we try to built the filter characteristics by cascading the characteristics of elementary discontinuities calculated between planes 2-3 and 4-5. The characteristics of discontinuities 5-6 and 7-8 are obtained by symmetry, while segments 1-2, 3-4, 6-7 and 8-9 are treated as homogeneous transmission lines with propagation constants produced by (13) when discontinuities 2-3 and 4-5 are analysed. In this approach, the calculated S-matrix elements of each segment change monotonously with frequency. With non-resonant character of the discontinuities, the FD-TD analysis of each segment is extremely fast. This does not disturb the process of extracting the parameters of the entire filter, and the cascaded results are practically the same as those in Fig.3. Maximum differences between S_{11} and S_{21} obtained by direct analysis of the entire filter, and by cascading the S-matrices of elementary discontinuities, are of the order of 0.005 in amplitude, plus a relative frequency shift up to 0.025%. Such inaccuracies are negligible in all practical applications.

IV. CONCLUSIONS

The proposed method proves to be very versatile, accurate and effective in practical applications. A combination of those properties permits to compare it favorably versus the previously reported methods. When considering the application of the presented method it is worth noting that:

- In the examples we have considered only one mode at each port, but the method works just as well in a multi-mode environment.
- The results presented above have all been extracted using mode templates calculated at a single frequency w_T =20.05 GHz. Very good results in the entire band are due to wide-band properties of the differential method which (by virtue of (12)) automatically takes into account the frequency dependent behaviour of the modal impedance. However, if inhomogeneously filled lines must be considered in a very wide band, the frequency dependence of the transverse field distributions $\boldsymbol{e}_{Ti}(x,y,w_T)$ and $\boldsymbol{h}_{Ti}(x,y,w_T)$ may cause some accuracy reduction of the modes' separation and absolute power reading. In such a case, it may be advisable to divide the band into 2-3 sub-bands and use different templates in each of the sub-bands.
- The mode template generation using 2-D time domain simulations like those described in [10] is limited to the cases when the absolute value of the required propagation constant of the evanescent mode is smaller than the absolute value of the propagation constant of the dominant mode at zero frequency. Otherwise [11] the 2-D time domain mode searching simulation becomes unstable and application of a frequency domain eigenvalue solver is recommended for the template generation phase.

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